North Carolina Department of Public Instruction
INSTRUCTIONAL SUPPORT TOOLS
FOR ACHIEVING NEW STANDARDS

Precalculus ● Unpacked Contents
For the new Standard Course of Study that will be effective in all North Carolina schools in the 2020-21 School Year.

This document is designed to help North Carolina educators teach the **Precalculus** Standard Course of Study. NCDPI staff are continually updating and improving these tools to better serve teachers and districts.

What is the purpose of this document?
The purpose of this document is to increase student achievement by ensuring educators understand the expectations of the new standards. This document may also be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. This document, along with on-going professional development, is one of many resources used to understand and teach the NC SCOS.

What is in the document?
This document includes a detailed clarification of each standard in the grade level along with a *sample* of questions or directions that may be used during the instructional sequence to determine whether students are meeting the learning objective outlined by the standard. These items are included to support classroom instruction and are not intended to reflect summative assessment items. The examples included may not fully address the scope of the standard. The document also includes a table of contents of the standards organized by domain with hyperlinks to assist in navigating the electronic version of this instructional support tool.

How do I send Feedback?
Link for: Feedback for NC's Unpacking Documents.
We will use your input to refine our unpacking of the standards. Thank You!

Just want the standards alone?
Link to: North Carolina Mathematics Standards

Precalculus Standards

Standards for Mathematical Practice

Number and Quantity	Algebra	Functions		
PC.N.1 *Apply properties of complex numbers and the complex number system.* PC.N.1.1 PC.N.1.2 ***PC.N.2*** *Apply properties and operations with matrices.* PC.N.2.1 PC.N.2.2 PC.N.2.3 PC.N.2.4 PC.N.2.5 ***PC.N.3*** *Understand properties and operations with vectors.* PC.N.3.1 PC.N.3.2	***PC.A.1*** *Apply properties of solving inequalities that include rational and polynomial expressions in one variable.* PC.A.1.1 PC.A.1.2 ***PC.A.2*** *Apply properties of solving equations involving exponential, logarithmic, and trigonometric functions.* PC.A.2.1 PC.A.2.2 PC.A.2.3 PC.A.2.4	***PC.F.1*** *Understand key features of sine, cosine, tangent, cotangent, secant and cosecant functions.* PC.F.1.1 PC.F.1.2 PC.F.1.3 PC.F.1.4 ***PC.F.2*** *Apply properties of a unit circle with center (0,0) to determine the values of sine, cosine, tangent, cotangent, secant, and cosecant.* PC.F.2.1 PC.F.2.2 ***PC.F.3*** *Apply properties of trigonometry to solve problems involving all types of triangles.* PC.F.3.1 PC.F.3.2 PC.F.3.3	***PC.F.4*** *Understand the relationship of algebraic and graphical representations of exponential, logarithmic, rational, power functions, and conic sections to their key features.* PC.F.4.1 PC.F.4.2 PC.F.4.3 PC.F.4.4 PC.F.4.5 PC.F.4.6 PC.F.4.7 PC.F.4.8 PC.F.4.9 ***PC.F.5*** *Apply properties of function composition to build new functions from existing functions.* PC.F.5.1 PC.F.5.2 PC.F.5.3 PC.F.5.4 PC.F.5.5 PC.F.5.6 PC.F.5.7	***PC.F.6*** *Apply mathematical reasoning to build recursive functions and solve problems.* PC.F.6.1 PC.F.6.2 ***PC.F.7*** *Apply mathematical reasoning to build parametric functions and solve problems.* PC.F.7.1 PC.F.7.2

Standards for Mathematical Practice

Practice	Explanation and Example
1. Make sense of problems and persevere in solving them.	In Precalculus (PC), students solve real world problems using their knowledge of numbers, functions, and algebra. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?”, “Does this make sense?”, and “Can I solve the problem in a different way?” Students also consider the reasonableness of intermediate results while applying processes to solve complex equations.
2. Reason abstractly and quantitatively.	In PC, students use algebraic, tabular, and graphical representations to reason about mathematical and real world contexts. They examine patterns in their processes. Students contextualize to understand the meaning of the number or variable as related to the problem. They mathematize problem situations to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others.	In PC, students construct arguments using verbal or written explanations accompanied by matrices, expressions, equations, functions, graphs, and tables. They further refine their mathematical communication skills through mathematical discussions in which they critically evaluate their own thinking and the thinking of other students. They construct arguments to defend functions they have created to model contextual situations. They pose questions like “How did you get that?”, “Why is that true?” “Does that always work?” They explain their thinking to others and respond to others’ thinking.
4. Model with mathematics.	In PC, students model problem situations symbolically, graphically, tabularly, and contextually. Students form expressions, generate functions, equations, or inequalities from real world contexts and connect symbolic and graphical representations. Students determine whether the model and the constraints they have constructed makes sense given the context of the problem.
5. Use appropriate tools strategically.	In PC, students consider available tools when solving mathematical problems and decide when particular tools might be helpful. It is assumed that students in PC will have access to graphing technologies (e.g., graphing calculator, Desmos) and spreadsheets. Students should examine results produced using technology to determine if the solution makes sense and be aware of issues that may arise when selecting an appropriate scale for a graph or mode to evaluate an expression. Students should also recognize when solving a problem by-hand is more efficient than a technology-assisted approach.
6. Attend to precision.	In PC, students use clear and precise language in their discussions with others and in their own reasoning. Students are aware of the effects of rounding on a solution. They recognize the importance and meaning of the symbols they use. They attend to units when solving real-world problems and appropriately label and interpret axes in graphs.
7. Look for and make use of structure.	Students routinely seek patterns or structures to model and solve problems. In PC, students reason about the solution and determine whether a simplified form is helpful for interpreting or using the result. Students are able to recognize key features of the graph of a function from its algebraic structure that may include asymptotes, end behavior, zeroes, amplitude, or period.
8. Look for and express regularity in repeated reasoning.	In PC, students use repeated reasoning to make generalizations about patterns and structures. By using repeated reasoning students are able to synthesize processes. For example, when examining a sequence of values students are able to develop a recursive function rule by identifying the repeated operations.
9. Use strategies and procedures flexibly.	In PC, students make choices in using algebraic procedures and selecting mathematical representations to solve problems. Students should recognize that an equation can be solved many ways which can include guess and check, algebraic, graphical, and tabular methods. Students should be able to use and make sense of different solution strategies and determine which approach may be most efficient. For example, students solving the inequality $3x^2 - 2x \leq 1$ may choose to solve this algebraically or graphically and recognize relationships between the two solution strategies.
10. Reflect on mistakes and misconceptions.	In PC, it is essential for students to reflect upon mistakes and misconceptions. Mistakes are often the cornerstone of learning. Successful students in this course will reflect upon their own thinking and learning to maximize their potential to find optimal solutions efficiently. For example, when students are generating a model of a situation, they may need to assess the model and make refinements so that it represents the contextual situation more accurately.

Return to: Standards

PC.N.1 Apply properties of complex numbers and the complex number system. PC.N.1.1 Execute the sum and difference algorithms to combine complex numbers.	
Clarification	**Checking for Understanding**
In Math 2 students use the quadratic formula to identify whether there are complex solutions to a quadratic equation. They express these complex solutions using *i.* Math 2 students have also learned that all real numbers are complex numbers (NC.M2.N-CN.1). In Precalculus students use properties to add and subtract complex numbers. They should recognize that the sum or difference of two complex numbers results in another complex number. Please note the sum and difference algorithms refer to adding and subtracting, respectively, the like terms, combining the real parts and combining the imaginary parts.	**Indicator:** Evaluate the following expressions: a. $(5+7i)+(-2+3i)$ b. $(9-i)-(4+6i)$ c. $(4-i\sqrt{2})-(3+i\sqrt{2})+8i$ *Answers: a. $3+10i$, b. $5-7i$, c. $1+(8-2\sqrt{2})i$*

Return to: Standards

PC.N.1 Apply properties of complex numbers and the complex number system. PC.N.1.2 Execute the multiplication algorithm with complex numbers.	
Clarification	**Checking for Understanding**
In Math 1 students multiply two binomial expressions. Students also rewrite expressions that involve complex numbers as described in PC.N.1.1. In Precalculus, students are expected to be able to multiply two complex numbers with and without technology. Please note the multiplication algorithm refers to multiplying the complex number by a real number, simply distributing as you would when multiplying polynomials. To multiply two complex numbers, you expand the product as you would with polynomials.	**Indicator**: Evaluate the following expressions: a. $2i(13-9i)$ b. $(3+4i)(8-5i)$ c. $2(2+4i)(-3+i)$ d. $-3(2i+1)\cdot(5i)$ *Answers: a. $18-26i$, b. $44+17i$ c. $-10-10i$, d. $30-15i$*

Return to: Standards

PC.N.2 Apply properties and operations with matrices. PC.N.2.1 Execute the sum and difference algorithms to combine matrices of appropriate dimensions.	
Clarification	**Checking for Understanding**
Precalculus is students' first experience with matrices. Students will need to be familiar with the elements of a matrix, be able to identify the rows, columns, and dimensions of a matrix. They should use dimensions to anticipate the dimensions of the sum and difference of two matrices and recognize that they can only add and subtract matrices that have the same dimensions. Students should also recognize that a matrix with all elements equal to zero is the identity element under addition and subtraction. Students are not expected to find the inverse of a matrix or solve problems involving contexts. Please note the sum and difference algorithms refer to adding and subtracting, respectively, matrices with the same dimensions. Students should do this with and without technology.	**Indicator:** Evaluate the following expressions: $M = \begin{bmatrix} -7 & a & b \\ 3 & 0 & 1 \end{bmatrix}$ and $J = \begin{bmatrix} 1 & 5 & b \\ 3 & 4 & 1 \end{bmatrix}$ a. Find $M + J$. b. Find $J - M$. *Answers:* *a.* $\begin{bmatrix} -6 & 5+a & 2b \\ 6 & 4 & 2 \end{bmatrix}$ *b.* $\begin{bmatrix} 8 & 5-a & 0 \\ 0 & 4 & 0 \end{bmatrix}$ **Indicator:** Given the matrix M, identify the matrix M_1, such that $M_1 \pm M$ will result in matrix M. $M = \begin{bmatrix} 5 & -3.6 & \frac{2}{7} \\ 3 & 21 & 0 \end{bmatrix}$ *Answer:* $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Return to: Standards

PC.N.2 Apply properties and operations with matrices. PC.N.2.2 Execute associative and distributive properties to matrices.	
Clarification	**Checking for Understanding**
Students learn and apply the associative and distributive properties to whole numbers in fifth grade. This is expanded to rational numbers, integers, and algebraic expressions in sixth and seventh grade. In Precalculus, students should apply their knowledge of the associative and distributive property to matrices. They should recognize matrices must have the same dimensions when adding and subtracting. Students should also be able to apply the distributive property when multiplying a scalar to the sum of two matrices	**Indicator:** Given the following matrices, evaluate the expressions and demonstrate how each could be used to show property of operations. a. $A + (B + C)$ b. $-2(A + B)$ $A = \begin{bmatrix} 2 & -3 \\ x & 1 \\ 4 & 7 \end{bmatrix}, B = \begin{bmatrix} 5 & y \\ -3 & 11 \\ 9 & -6 \end{bmatrix}$, and $C = \begin{bmatrix} -8 & 1 \\ -2 & -5 \\ z & 10 \end{bmatrix}$ *Answers:* *a.* $\begin{bmatrix} -1 & y-2 \\ x-5 & 7 \\ 13+z & 11 \end{bmatrix}$ *b.* $\begin{bmatrix} -14 & -2y+6 \\ -2x+6 & -24 \\ -26 & -2 \end{bmatrix}$ *Demonstrate the associative property by showing that A+(B+C) =* $(A + B) + C$ *Demonstrate the distributive property by showing that -2(A+B)=* $-2A - 2B$

Return to: Standards

PC.N.2 Apply properties and operations with matrices. PC.N.2.3 Execute commutative property to add matrices.	
Clarification	**Checking for Understanding**
Students learn and apply the commutative property to whole numbers in fifth grade. This is expanded to rational numbers, integers, and algebraic expressions in sixth and seventh grade. In Precalculus, students should apply their knowledge of the commutative property to add matrices. They should recognize that subtraction of matrices is not commutative and know that they can only add matrices that have the same dimensions.	**Indicator:** Using the following matrices, demonstrate the commutative property of addition. $A = \begin{bmatrix} 2 & -3 \\ x & 1 \\ 4 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & y \\ -3 & 11 \\ 9 & -6 \end{bmatrix}$ *Answer: The students should show that $A + B = B + A$. Both sums should be* $\begin{bmatrix} 7 & y-3 \\ x-3 & 12 \\ 13 & 1 \end{bmatrix}$.

Return to: Standards

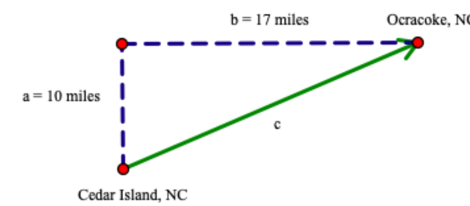
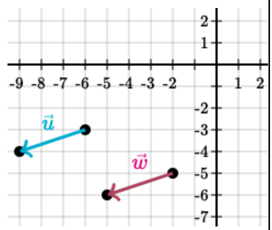
PC.N.2 Apply properties and operations with matrices. PC.N.2.4 Execute properties of matrices to multiply a matrix by a scalar.	
Clarification	**Checking for Understanding**
In Precalculus, students should be able to multiply a matrix by a scalar and use dimensions to anticipate the dimensions of the product.	**Indicator:** Use the given matrices to evaluate the following expressions. $N = \begin{bmatrix} 10 & 0 & 11 \\ -2 & 3 & -1 \\ 5 & -25 & 4 \end{bmatrix}$ and $K = \begin{bmatrix} -7 & 1 & 9 \\ 3 & 0 & 1 \end{bmatrix}$ a. Find $-3N$ b. Find $\sqrt{3}K$ *Answers: a.* $\begin{bmatrix} -30 & 0 & -33 \\ 6 & -9 & 3 \\ -15 & 75 & -12 \end{bmatrix}$, *b.* $\begin{bmatrix} -7\sqrt{3} & \sqrt{3} & 9\sqrt{3} \\ 3\sqrt{3} & 0 & \sqrt{3} \end{bmatrix}$

Return to: Standards

PC.N.2 Apply properties and operations with matrices. PC.N.2.5 Execute the multiplication algorithm with matrices.	
Clarification	**Checking for Understanding**
Students should recognize that in order to multiply two matrices the number of columns in the first matrix must be equal to the number of rows in the second matrix. They should reason about the dimensions of matrices to determine the dimension of the product. Students should also be able to determine the identity element for matrix multiplication. They are not expected to solve matrix equations. Students should recognize that matrix multiplication is associative, but not commutative. Students should be able to multiply matrices with and without technology.	**Indicator:** Without the aid of technology, what is element $e_{23}$of the product of $G \cdot H$? $\boldsymbol{G} = \begin{bmatrix} \mathbf{3} & \mathbf{2} \\ \mathbf{5} & \mathbf{0.5} \end{bmatrix}$ $\boldsymbol{H} = \begin{bmatrix} \mathbf{7} & \mathbf{-1} & \mathbf{0.5} \\ \mathbf{-5} & \mathbf{-3.5} & \mathbf{4} \end{bmatrix}$ *Answer: 4.5*

Please note the multiplication algorithm with matrices refers to multiplying two matrices as explained above.	**Indicator:** Use the following matrices to find $B \cdot A$. $$A = \begin{bmatrix} 10 & 0 & 11 \\ -2 & 3 & -1 \\ 5 & -25 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} -7 & a & b \\ 3 & 0 & 1 \end{bmatrix}$$ *Answers:* $\begin{bmatrix} -70-2a+5b & 3a-25b & -77-a+4b \\ 35 & -25 & 37 \end{bmatrix}$ **Indicator:** Given matrix M, identify the matrix M_1 and M_2, such that: $M = \begin{bmatrix} 5 & -3.6 & \frac{2}{7} \\ 3 & 21 & 0 \end{bmatrix}$ a. $M_1 \cdot M = M$ b. $M \cdot M_2 = M$ *Answers: a.* $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, *b.* $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Return to: Standards

PC.N.3 Understand properties and operations with vectors. PC.N.3.1 Represent a vector indicating magnitude and direction.	
Clarification	**Checking for Understanding**
Students learn how to plot points in the coordinate plane in 6th grade and learn the Pythagorean Theorem in 8th grade. Precalculus students can build on this knowledge to represent vectors in the coordinate plane using component form only. They describe vectors using magnitude and direction. Students can apply the Pythagorean theorem to find the magnitude of a vector and can use angles to describe the direction of a vector by describing its bearing (examples: 30 degrees north of east, 20 degrees west of south). They know two vectors are equivalent if they have the same magnitude and direction. Students are not expected to execute procedures for vectors beyond 2-dimensional vectors.	**Indicator:** A ferry departs Cedar Island, NC and travels to Ocracoke, NC which is 10 miles north and 17 miles east from the point of departure. Use a vector to represent the path of the ferry and determine how far it traveled.  Answer: $< 17{,}10 >$, To find the distance the ferry traveled, find the magnitude of the vector. $c = 19.72\ miles$ **Indicator:** Determine whether vectors $\boldsymbol{u}$ and $\boldsymbol{w}$ are equivalent  *Answer: The vectors are equivalent because they have the same magnitude and direction (because their slopes are the same)* **Indicator:** A pilot flies a plane that takes off from an airport and travels due west for 150 miles. The pilot then turns due north and travels 150 miles. Using vectors, how would you find the distance and direction of the plane from the airport. *Answer: The distance is the magnitude of the resultant vector,* $< -150{,}150 >$.

	The magnitude is approximately 212.13 miles. *The direction can be found logically or using the triangle formed by the vectors and the resultant. The direction can be described in a variety of ways, such as 45 °north of west, or directly northwest.*

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<table>
<tr><td colspan="2">PC.N.3 Understand properties and operations with vectors.
PC.N.3.2 Execute sum and difference algorithms to combine vectors.</td></tr>
<tr><td>Clarification</td><td>Checking for Understanding</td></tr>
<tr><td>

Students have not previously learned how to add and subtract vectors. In Precalculus students should be familiar with three different methods for adding and subtracting vectors:

- Adding vectors end-to-end - positioning the vectors (without changing their magnitudes and directions) so that the initial point of one vector coincides with the terminal point of the other vector
- Adding/subtracting corresponding components - add or subtract the corresponding components
- Using the parallelogram rule - a graphical method used for:
  - addition of two vectors,
  - subtraction of two vectors, and
  - resolution of a vector into two components in arbitrary directions.

Students should understand the process of finding the sum/difference of two vectors using any of the methods mentioned. They are not expected to know the name of the methods.
Students are not expected to execute procedures for vectors beyond 2-dimensional vectors.

Please note the sum and difference algorithms refer to adding and subtracting, respectively, vectors using the methods explained above. Students are not expected to know how to compute the dot product or find a unit vector.

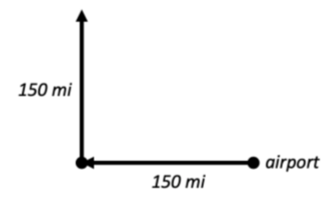
</td><td>

Indicator: Given the vectors $u =< -10,12 >$ and $w =< 5,-10 >$

a. Find $u + w$.
b. Find $u - w$.

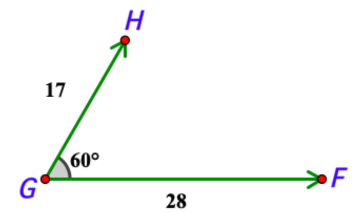
Answers: a. $< -5,2 >$, b. $< -15,22 >$

Indicator: A pilot flies a plane that takes off from an airport and travels due west for 150 miles. The pilot then turns due north and travels 150 miles. Use vectors to describe the location of the plane from the airport.

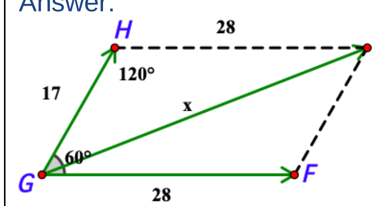


Answer: $< -150,0 > + < 0,150 > = < -150,150 >$

Indicator: Given vector GH and vector GF with an angle of 60 degrees between them, find the magnitude of the resultant vector.



Answer:



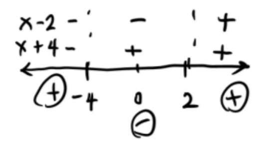
$$x^2 = 17^2 + 28^2 - 2(17)(28) * cos(120)$$
$$x = 39.36$$

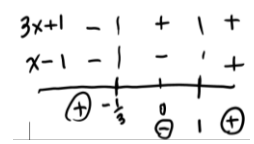
</td></tr>
</table>

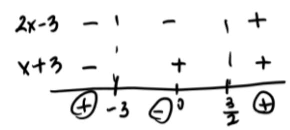
Return to: [Standards]

<table>
<tr><td colspan="2">PC.A.1 Apply properties of solving inequalities that include rational and polynomial expressions in one variable.
PC.A.1.1 Implement algebraic (sign analysis) methods to solve rational and polynomial inequalities.</td></tr>
<tr><td>Clarification</td><td>Checking for Understanding</td></tr>
<tr><td>In Math 1, students solve linear inequalities in one variable. In Math 3, students create inequalities in one variable that represent absolute value, polynomial, exponential, and rational functions.

In Math 3, students solve rational and polynomial equations and are expected to represent the solutions of an inequality using a number line and compound inequalities using inequality and interval notation.

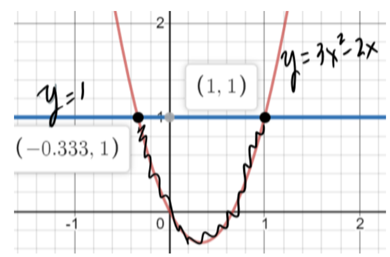
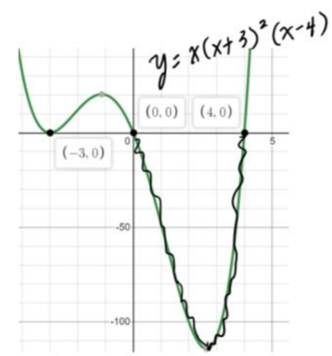
In Precalculus, students build on these concepts by creating and solving inequalities in order to find the domain of various functions involving rational and factorable polynomial functions and stating the domain of the function using interval notation. Solutions include only real numbers. They also solve more complex inequalities outside of finding domains.</td><td>Formative Check: Use sign analysis to identify the domain of f where $f(x) = \sqrt{\frac{x-2}{x+4}}$. Use interval notation to describe your answer.
Answer:
$\frac{x-2}{x+4} \geq 0$
Domain: $(-\infty, -4] \cup [2, \infty)$


Formative check: Use algebraic methods and sign analysis to find the domain of the function $f(x) = \sqrt{3x^2 - 2x - 1}$. Use interval notation to describe your answer.
Answer:
$(3x+1)(x-1) \geq 0$
Domain: $(-\infty, -\frac{1}{3}] \cup [1, \infty)$


Formative check: Solve $2x^4 < -3x^3 + 9x^2$ using algebraic methods and sign analysis. Use interval notation to describe your answer.
Answer:
$x^2(2x-3)(x+3) < 0$
Solution: $\left(-3, \frac{3}{2}\right)$


Indicator: A ball is thrown vertically upward with an initial velocity of 80 feet per second. In the formula $s = 80t - 16t^2$, s is the height of the ball from the ground in feet after t seconds. For what interval is the ball more than 96 feet above ground?
Answer: 2 < t < 3</td></tr>
</table>

Return to: [Standards](#)

PC.A.1 Apply properties of solving inequalities that include rational and polynomial expressions in one variable. PC.A.1.2 Implement graphical methods to solve rational and polynomial inequalities.	
Clarification	**Checking for Understanding**
In Math 3 students factor and solve equations involving higher-order polynomials. They extend an understanding that the x-coordinates of the points where the graphs of two equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$ and approximate solutions using a graphing technology. In Precalculus, students apply their knowledge of factoring quadratics and higher-order polynomials and extend this knowledge using technology to solve inequalities.	**Indicator:** Use graphical methods to solve the inequalities using interval notation to describe your answer. a. $3x^2 - 2x \leq 1$ b. $x(x+3)^2(x-4) < 0$. *Answers:*  *a. Using the graph to the left, we see that the parabola is less than or equal to $y = 1$ for x's in the interval [-⅓,1] or [-0.33,1].*  *b. Using the graph to the left, we can see that the 4th-degree polynomial is strictly less than 0 for x-values in the interval (0,4).*

Return to: Standards

PC.A.2 Apply properties of solving equations involving exponential, logarithmic, and trigonometric functions. PC.A.2.1 Use properties of logarithms to rewrite expressions.	
Clarification	**Checking for Understanding**
In Math 3, students use logarithms to solve equations such as $ab^{ct} = d$, where *a*, *b* and *c* are real numbers. In Precalculus, students will extend their knowledge of properties of logarithms to rewrite expressions involving logarithms.	**Indicator:** Rewrite the following expressions as a single term: a. $log(x-3) + log(x+4)$ b. $log(x-3) - log(x+4)$ c. $3log(x) - log(x-2)$ d. $2log(x+1) + 3log(x)$ *Answers: a.* $log((x-3)(x+4))$, *b.* $log\left(\frac{x-3}{x+4}\right)$ *c.* $log\left(\frac{x^3}{x-2}\right)$ *d.* $log\left(x^3\,(x+1^2)\right)$

Return to: Standards

PC.A.2 Apply properties of solving equations involving exponential, logarithmic, and trigonometric functions. PC.A.2.2 Implement properties of exponentials and logarithms to solve equations.	
Clarification	**Checking for Understanding**
In Math 3, students solve exponential equations with common bases and logarithmic equations with common bases. In Precalculus, students will extend their knowledge of properties of logarithms to rewrite expressions to solve problems involving logarithms that include the sum, difference and power rules.	**Formative check:** Solve the equation for *x*. Give the exact solution(s). $log(x-3)+\ log(x+4)=2log3$ *Answer:* $x=\frac{-1+\sqrt{85}}{2}$ **Formative check:** Find the exact solution for $5^{x-3}=2^{2x}$. *Answer: Solution methods may vary. Sample solution:* $x=\frac{3}{1-(2log_5 2)}$ **Indicator:** A radioactive substance decays at an exponential rate and the amount of the substance, $A(t)$, can be modeled using the following function: $A(t)=A_0e^{-0.05t}$ where *t* is measured in days and A_0 is the initial amount of the substance. If we start with 1000 grams of the substance, how long will it take for 800 grams of the substance to decay? Round your answer to the nearest tenth of a day. *Answer: 32.2 days*

Return to: Standards

PC.A.2 Apply properties of solving equations involving exponential, logarithmic, and trigonometric functions. PC.A.2.3 Implement properties of trigonometric functions to solve equations including • inverse trigonometric functions, • double angle formulas, and • Pythagorean identities.	
Clarification	**Checking for Understanding**
In Math 3, students should build an understanding of the unique relationship between the measure of the angle and the value of the particular trigonometric ratio. Also in Math 3, students build an understanding of radian measure. NC.M3.F-TF.1 In Math 3, students also began to see the graphical representations of trigonometric functions, both on a unit circle and on a graph in which the domain is the measure of the angle and the range is the value of the associated trigonometric ratio. On the unit circle, the input is the measure of the angle and the output of the sine function is the y- coordinate of the vertex of the formed triangle. In Precalculus, students will extend their knowledge of trigonometric equations, by learning trigonometric identities and using those identities and algebra to solve trigonometric equations. They are only expected to	**Formative check:** Solve $cos\,(2\theta)+6sin^2\theta=4$ on the interval $0\leq\theta<2\pi$. *Answer:* $\theta=\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3},\frac{5\pi}{3}$ **Indicator:** The depth of the water at the end of a pier varies with the tides throughout the day. This phenomenon can be modeled by the function $D(t)=2.1cos\left(\frac{\pi}{5.75}(t-4.5)\right)+5.1$, where D represents the depth of the water in feet and t is hours after midnight. What is the first time after midnight when the depth of the water is 6 feet? *Answer: Around 2:27 AM*

know the Pythagorean and double angle identities. Students are not expected to prove trigonometric identities. Students will build on their knowledge of the exact values of trigonometric functions for special angles (from Math 3) to find the exact values of solutions to solve these more complex trigonometric equations. These equations can include all six trigonometric ratios. Students will not be expected to solve for all possible solutions with a phase shift and period change.	

Return to: Standards

PC.A.2 Apply properties of solving equations involving exponential, logarithmic, and trigonometric functions. PC.A.2.4 Implement algebraic techniques to rewrite parametric equations in cartesian form by eliminating the parameter.	
Clarification	**Checking for Understanding**
This builds on algebraic knowledge of solving for one variable in terms of other variables in Math 1. In Precalculus, the concept of parametric equations is new and students are asked to connect the idea of expressing a curve in the plane in terms of x and y to the concept of defining a curve in the plane by breaking that curve up into its x(t) and y(t) representations in terms of the parameter t. Students are not expected to convert from rectangular to parametric form.	**Indicator:** Two planes depart from an airport and their path is observed by air traffic control on a monitor. The first plane's path can be described using the parametric equations $x(t) = t + 4$ and $y(t) = 3t - 1$. The second plane's path can be described by the parametric equations $x(t) = t + 4$ and $y(t) = 2t + 9$. a. Eliminate the parameter for the equations given for the first plane and express its path as a function, $y = f(x)$. b. Eliminate the parameter for the equations given for the second plane and express its path as a function, $y = f(x)$. c. Find the x and y coordinates of when the paths intersect. Will the planes collide? Defend your answer mathematically. *Answers: a. $y = 3x - 13$, b. $y = 2x + 1$* *c. The two paths will intersect at the point (14,29).* *The two paths will intersect when the equations intersect.* *When x is 14, both equations will be 29.* $3x - 13 = 2x + 1$ $x = 14$ $y = 2(14) + 1 = 29$

Return to: Standards

PC.F.1 Understand key features of sine, cosine, tangent, cotangent, secant and cosecant functions.

PC.F.1.1 Interpret algebraic and graphical representations to determine key features of transformed sine and cosine functions. *Key features include: amplitude, domain, midline, phase shift, frequency, period, intervals where the function is increasing, decreasing, positive or negative, relative maximums and minimums.*

Clarification

In Math 3, students were introduced to the concept of a periodic graph through the sine function and the effects of the various representations by changing parameters, a, b, and h of a sine function f(x) = a sin (bx) + h. Students used technology to investigate these changes. Phase shifts are not part of the standards for Math 3. (NC.M3.F-TF.5)

In Math 3, period, amplitude, domain, range, and midline were the key features studied. Phase shift and frequency are new terms for students in Precalculus.

In Precalculus, students will build on their knowledge of the parameters and interpreting key features of trigonometric functions. An extension is made from NC Math 3 to transformations of the sine and cosine functions in Precalculus. Tangent, cotangent, secant and cosecant are studied as parent functions.

In Precalculus, students will build on their knowledge of the parameters and interpreting key features of trigonometric functions.
The standard for sine and cosine functions:

$$f(x) = a\sin(b(x+c)) + d$$
$$g(x) = a\cos(b(x+c)) + d$$

Students will use a, b, c, and d for the parameters of the functions where a = amplitude, b = frequency (the number of cycles completed in a given interval), c = horizontal shift and d is the vertical shift.
There is no restriction on the domain unless otherwise stated in the problem(s). Students should demonstrate an understanding that while vertical shifts alter the maximum and minimum values of a function, they do not alter the amplitude. Also, horizontal shifts do not affect the amplitude.

Checking for Understanding

Indicator: For $f(x) = 4\sin(2(x-\frac{\pi}{2})) + 5$ identify the period, frequency, amplitude, domain, range, midline, and phase shift.
Answers:

period: π *frequency = 1/π* *amplitude: 4*

domain: $(-\infty, \infty)$ *range: [1, 9]* *midline: $y = 5$*

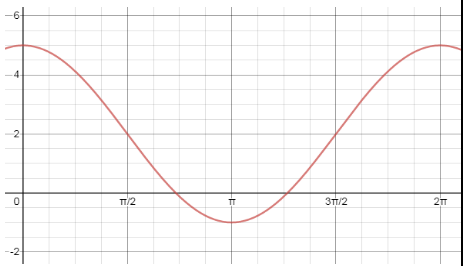
phase shift: $\pi/2$

Indicator: Consider the function $f(x) = \sin(2x)$ on the domain $[0,2\pi]$

give the values of x in the given interval for which f is increasing.
Answer: $\left(0,\frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4},\frac{5\pi}{4}\right) \cup \left(\frac{7\pi}{4},2\pi\right)$

Indicator: Consider the function $f(x) = \sin(x+\frac{\pi}{4})$on the domain $[0,2\pi]$, give the values of x in the given interval for which $f(x)$ is greater than zero.
Answer: $\left(0,\frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4},2\pi\right)$

Indicator: Given the graph, identify the amplitude, phase shift, midline, maximum, minimum, and period of the function.



Answer:
If student sees the graph as a cosine function:

amplitude: 3 *phase shift: NA* *midline: y = 2* *maximum = 5*
minimum = -1 *period: 2π*

If student sees the graph as a sine function:

amplitude: 3 *phase shift: $\frac{\pi}{2}$* *midline: $y = 2$* *maximum = 5*
minimum = -1 *period: 2π*

Return to: Standards

PC.F.1 Understand key features of sine, cosine, tangent, cotangent, secant and cosecant functions.

PC.F.1.2 Interpret algebraic and graphical representations to determine key features of tangent, cotangent, secant, and cosecant. *Key features include: domain, frequency, period, intervals where the function is increasing, decreasing, positive or negative, relative maximums and minimums, and asymptotes.*

Clarification

In Math 3, students examine more complex functions represented by graphs and tables and focus on interpreting key features of all function types; there are no limitations on function types. Students learned about periodicity as motion that is repeated in equal intervals of time and discontinuity as values that are not in the domain of a function, either as asymptotes or “holes” in the graph. Students did not have to be able to algebraically manipulate a function in order to identify the key features found in graphs, tables, and verbal descriptions. (NC.M3.F-IF.4)

In Math 3, key feature concepts are extended to the sine and cosine functions. Discontinuity (asymptotes/holes) and periodicity were introduced. Students found discontinuities in tables and graphs and recognized their relationship to functions. The intent for Math 3 students is to find discontinuities in tables and graphs and to recognize their relationship to functions.

In Precalculus, students will determine and interpret key features previously studied to include the vertical asymptotes for tangent, cotangent, secant, and cosecant functions. Students are not expected to find asymptotes from the algebraic expression of a trigonometric function. Students are only expected to describe phase shift and change of period for sine and cosine.

Checking for Understanding

Indicator: Consider $f(x) = csc(x)$ on the interval $[0,2\pi]$.

a. Give the equations of the vertical asymptotes for f in the interval.
b. Give the values of x for which f is positive.

Answers: a. $x = 0, x = \pi, x = 2\pi$, *b.* f *is positive on* $(0,\pi)$

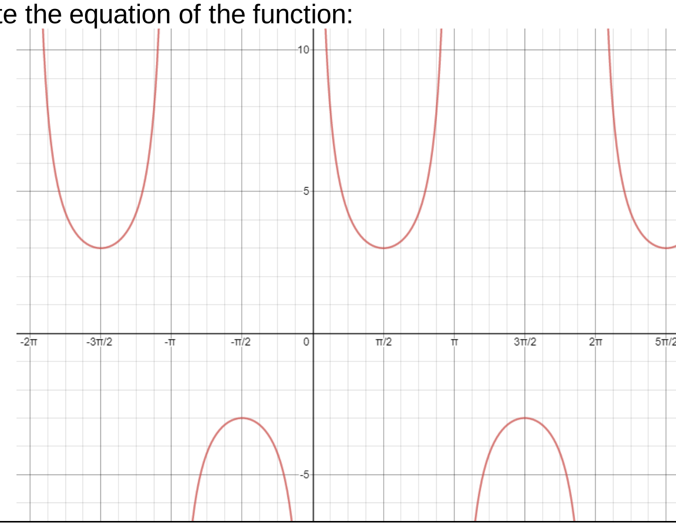
Indicator: Consider $f(x) = tan(x)$ on the interval$[0,2\pi]$.

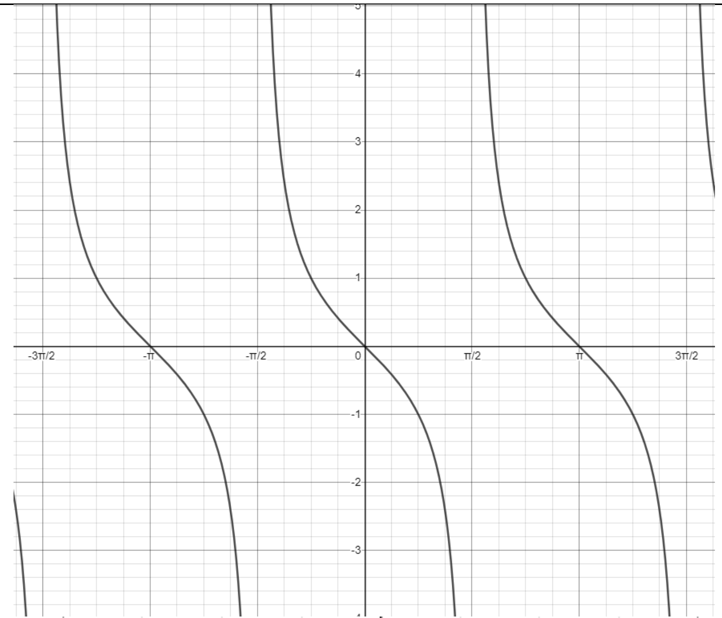
a. Give the equations of the vertical asymptotes for f in the interval.
b. Give the values of x for which f is positive.

Answer: a. $x = \frac{\pi}{2}, x = \frac{3\pi}{2}$, *b.* $f(x) > 0$ *on* $(0,\frac{\pi}{2}) \cup \left(\pi,\frac{3\pi}{2}\right)$

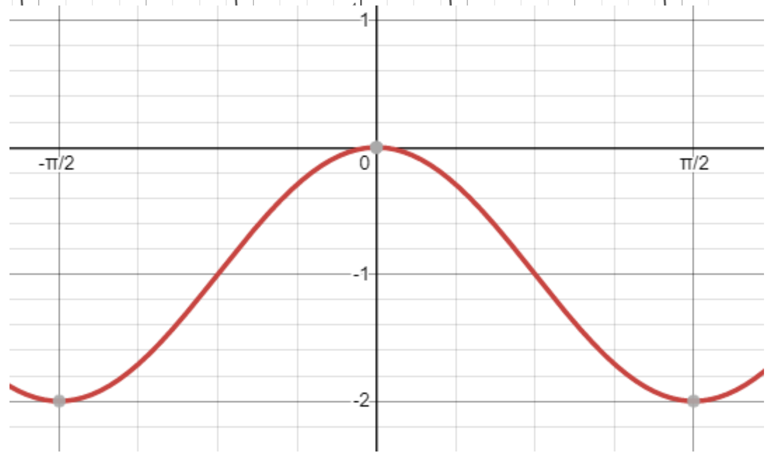
Indicator: Given the following graphs, use your understanding of key features of trigonometric functions to:

i. Give the first positive interval where f(x) > 0 for one period
ii. Give the equations of the vertical asymptotes for the interval [-π, π]
iii. Give an expression for the domain of the function
iv. Write the equation of the function:

a. 



b.



c.

Answers:

a. i. $(\frac{\pi}{2}, \pi)$ *ii.* $x = -\frac{\pi}{2}, x = \frac{\pi}{2}$ *iii. D:* $x \epsilon \Re$, $x \neq \frac{k\pi}{2}$ *iv.* $y = 3csc(x)$

b. i. (0, π/2) *ii.* $x = \frac{k\pi}{2}$ *iii. D:* $x \epsilon \Re$, $x \neq \frac{k\pi}{2}$ *iv.* $y = -tan(x)$

c. i. never ϕ *ii. none* *iii. (-∞, ∞)* *iv.* $y = sin(2x + \frac{\pi}{2}) - 1$

Return to: Standards

PC.F.1 Understand key features of sine, cosine, tangent, cotangent, secant and cosecant functions.

PC.F.1.3 Integrate information to build trigonometric functions with specified amplitude, frequency, period, phase shift, or midline with or without context.

Clarification

In Precalculus, students make the connection of the constant *a* of *f(x) = a sin x and f(x) = a cos x* of the trigonometric functions to amplitude, the distance from the midline to a maximum or minimum, for the sine and cosine functions. They will use their understanding of one period as one cycle over a given distance and the number of cycles within a specified domain is the frequency.

In Precalculus, students will use the amplitude, frequency, period, phase shift, and/or midline to create a function in the standard form for a context or a graph.

Checking for Understanding

Indicator: Build a sine or cosine function that has a period π an amplitude of 4 and phase shift of $\pi/2$.

Answer: $f(x) = 4\cos(2x - \pi)$

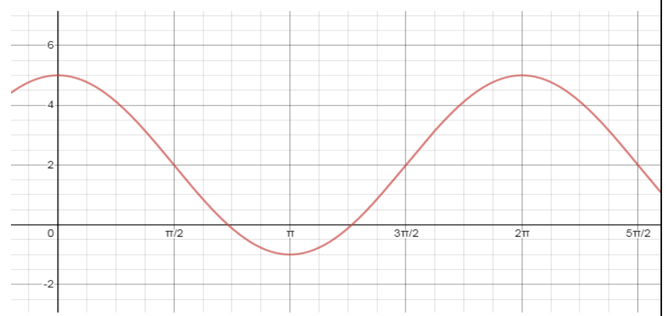
Indicator: A person is riding on a Ferris wheel and it takes 2 minutes for the wheel to make one complete revolution. The person's vertical distance from the ground ranges between 3 feet to 75 feet. Suppose at time $t = 0$, the person is at the minimum height of 3 feet. Build a model using sine or cosine that describes the person's vertical distance from the ground over time, where height is measured in feet and time is measured in minutes.

Answer: $f(x) = -[36\cos(\pi x)] + 39$

Indicator:

Using the graph:

a. Identify the amplitude, phase shift, midline, and period of the function.
b. Write an equation for the function.



Answers:

If student sees the graph as a cosine function:

a. amplitude: 3 phase shift: NA midline: $y = 2$ period: 2π

b. $f(x) = 3\cos(x) + 2$

If student see the graph as a sine function:

a. amplitude: 3 phase shift: $\frac{\pi}{2}$ midline: $y = 2$ period: 2π

b. $f(x) = 3\sin\left(x + \frac{\pi}{2}\right) + 2$

Return to: [Standards]

PC.F.1 Understand key features of sine, cosine, tangent, cotangent, secant and cosecant functions. PC.F.1.4 Implement graphical and algebraic methods to solve trigonometric equations and inequalities in context with support from technology.	
Clarification	**Checking for Understanding**
In Math 2, Use trigonometric ratios and the Pythagorean Theorem to solve problems involving right triangles in terms of a context. (NC.M2.G-SRT.8) In Math 3, students analyze trigonometric functions (sine and cosine) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, or negative; rate of change; relative maximums and minimums; symmetries; end behavior; period; and discontinuities. (NC.M3.F-IF.7) In Precalculus, the study of key features of trigonometric functions is extended to include tangent, cotangent, secant, and cosecant functions. Students will use their understanding of algebra and key features of trigonometric functions to evaluate and solve equations and inequalities with or without technology.	**Indicator:** A person is riding on a Ferris wheel and it takes 2 minutes for the wheel to make one complete revolution. The function $h(t) = -31cos(\pi \cdot t) + 65$gives the person's height off the ground (measured in feet) t minutes after the ride begins. a. What is the maximum height of the person on the Ferris wheel? b. If the person rides on the Ferris wheel for 10 minutes, when is the person's height above the ground greater than 60 feet? *Answers:* *a. 96 ft,* *b.* $t = (0.45, 1.55) \cup (2.45, 3.55) \cup (4.55, 5.55) \cup (6.45, 7.55) \cup (8.45, 9.55)$

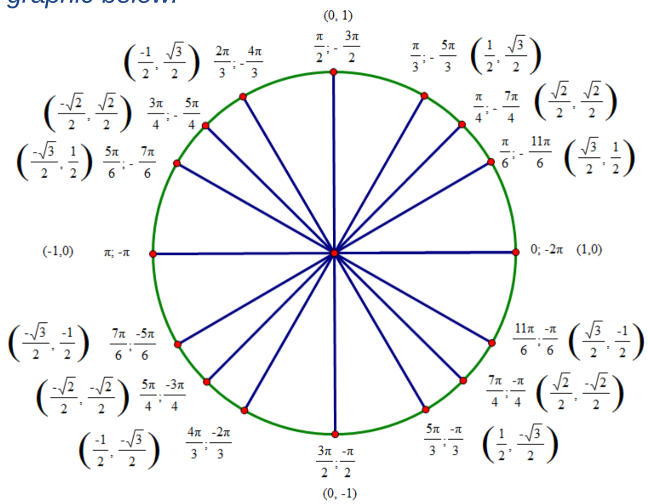
Return to: Standards

PC.F.2 Apply properties of a unit circle with center (0,0) to determine the values of sine, cosine, tangent, cotangent, secant, and cosecant. PC.F.2.1 Use a unit circle to find values of sine, cosine, and tangent for angles in terms of reference angles.	
Clarification	**Checking for Understanding**
In Math 3, students build an understanding of trigonometric functions by using tables, graphs and technology to represent the cosine and sine functions. Interpret the sine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its *y* coordinate.(NC.M3.F-TF.2a) and interpret the cosine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its *x* coordinate (NC.M3.F-TF.2b). In Math 3, students are only introduced to the trigonometric functions and build upon previous understanding of the trig relationships in right triangles. Students are introduced to the unit circle and angle measures on the coordinate plane in Math 3 as a way to relate the sine and cosine ratios to the coordinates and the plane. An in-depth teaching of the unit circle, tangent and reciprocal ratios, coterminal angles, specific	**Indicator:** Given $sin(x) = -\frac{1}{3}$, find the values of: a. $cos(x)$ b. $sin(x - \pi)$ c. $tan(x)$ *Answers: a.* $\pm\frac{2\sqrt{2}}{3}$, *b.* $\frac{1}{3}$, c. $\pm\frac{1}{2\sqrt{2}}$

coordinates and the Pythagorean Identity are NOT part of the standards for Math 3. In Precalculus, students use the unit circle to determine the value of trig functions in any quadrant using the reference angle. They understand the absolute value of the sine and cosine of an angle in any quadrant is the same as its reference angle; the sign depends on the quadrant of the original angle. Students learn to use a trigonometric ratio to find the value of a tangent function.	

Return to: Standards

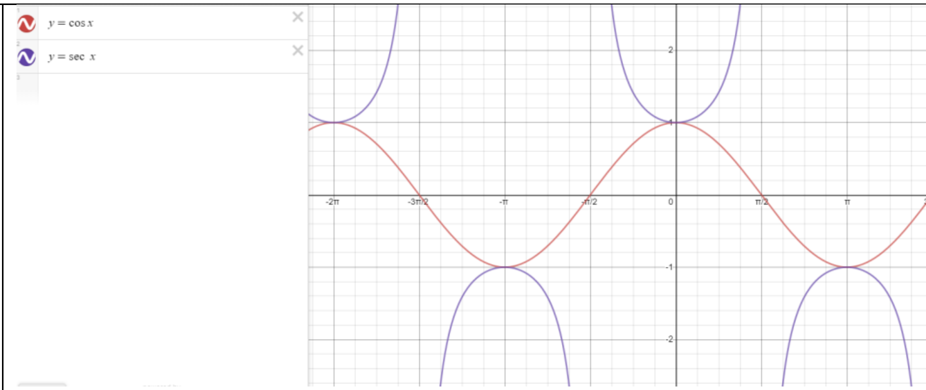
PC.F.2 Apply properties of a unit circle with center (0,0) to determine the values of sine, cosine, tangent, cotangent, secant, and cosecant.
PC.F.2.2 Explain the relationship between the symmetry of a unit circle and the periodicity of trigonometric functions.

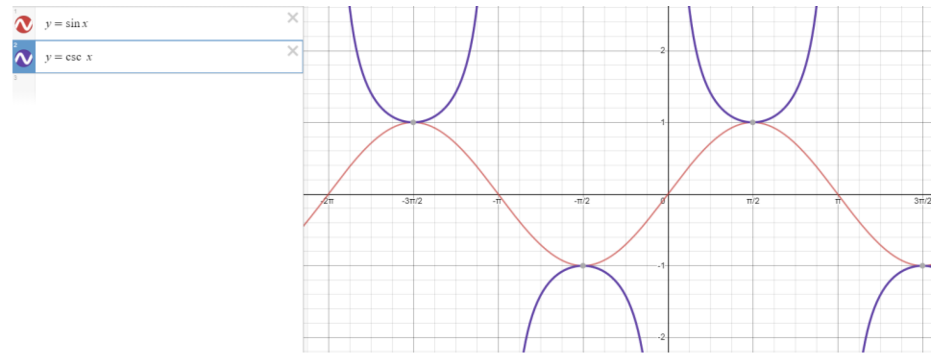
Clarification	**Checking for Understanding**
In Math 2, students learn to identify center and angle(s) of rotation symmetry and identify line(s) of reflection symmetry. (NCM2.G-CO.3). Students begin to make the connection of the even and odd of a geometric figure and its properties. In Math 3, students learn to interpret the sine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its y coordinate. They also learn to interpret the cosine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its x coordinate. (NC.M3.F-TF.2) Students are introduced to the unit circle and angle measures on the coordinate plane in Math 3 as a way to relate the sine and cosine ratios to the coordinates and the plane. (NC.M3.F-TF.2b). Math 3 provides an introduction to the concept of a periodic graph through learning the sine function. (NCM3.F-TF.5) In Precalculus, students will develop an understanding of using the trigonometric ratios and the coordinates of the unit circle to determine the values of sine, cosine, tangent, and their reciprocal functions. Students will extend on the lines of reflection symmetry and angle of rotation symmetry previously studied to symmetry with respect to the y-axis and symmetry with respect to the origin to identify the symmetries and even/odd functions. Students should make the connection that a function has symmetry with respect to the y-axis if $(x, y)$and $(-x, y)$are	**Indicator:** Explain why $csc(-\theta) = -csc(\theta)$and $sec(-\theta) = sec(\theta)$. Do these equations hold for any angleθ? Demonstrate your reasoning using a unit circle and the graph of the trigonometric function. *Answer: Students should use their knowledge of trig functions to demonstrate the pattern within the cosecant and secant functions. Students should build a unit circle using common values and focus on the symmetries produced, as seen in the graphic below.*  *Students should create graphs of the trig functions as seen below. In the graphs, the symmetries and reflections noted on the circle graph can now be seen visually. Students should be able to explain these connections. Students should also be able to explain how the patterns continue beyond their representations.*

both coordinates of the function on the unit circle or $f(-x) = f(x)$that $f(x)$is an even function. Similarly, a function that has symmetry with respect to the origin if $(x, y)$and $(-x, -y)$ are both coordinates of the function on the unit circle or $f(-x) = -f(x)$ that $f(x)$is an odd function.

Students will use their understanding of a period of a sine function to extend to periods for cosine, tangent, cotangent, secant, and cosecant. Students should make the connection of lines of symmetry, repeating coordinates on the unit circle, and periods of the functions.

The periodicity of trigonometric functions shows the identities:
$sin\ (\alpha + k \cdot 2\pi) = sin\ (\alpha)$ and $cos\ (\alpha + k \cdot 2\pi) = cos\ (\alpha)$, $k\ \epsilon\ Z$
$tan\ ((\alpha + k \cdot \pi) = tan\ (\alpha)$ and $cot\ (\alpha + k \cdot \pi) = cot\ (a)$, $k\ \epsilon\ Z$





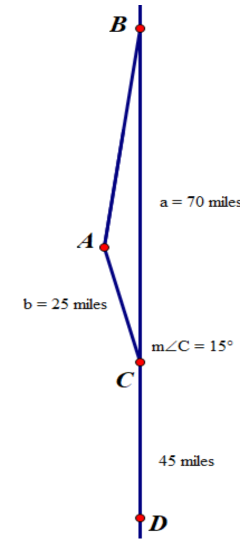
Indicator: Explain why $tan(\pi + \theta) = tan(\theta)$and $sec(2\pi + \theta) = sec(\theta)$. Do these equations hold for any angleθ? Explain.

Answer: The period for the tangent function is πradians and the coordinates repeat on the unit circle every period, therefore $tan(\pi + \theta) = tan(\theta)$. Similarly, the period for a secant function is 2π radians and the coordinates repeat on the unit circle every period, therefore $sec(2\pi + \theta) = sec(\theta)$.

Return to: Standards

PC.F.3 Apply properties of trigonometry to solve problems involving all types of triangles. PC.F.3.1 Implement a strategy to solve equations using inverse trigonometric functions.	
Clarification	**Checking for Understanding**
In Math 2, students connect proportional reasoning from 7th grade to work with right triangle trigonometry. They are able to use trigonometric ratios and the Pythagorean Theorem to solve problems involving right triangles in terms of a context. Students use the Pythagorean Theorem to develop relationships between the sides of a 30°-60°-90° triangle and develop and justify relationships between the sides of a 45°-45°-90°triangle. (NCM2.G-SRT-8 and NCM2.G-SRT.12) In Math 3, Students build an understanding of trigonometric functions by using tables, graphs and technology to represent the cosine and sine functions. They Interpret the sine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its *y* coordinate and the cosine function as the relationship between the radian measure of an angle formed by the horizontal axis and a terminal ray on the unit circle and its *x* coordinate. Students develop an understanding of radian angle measure and applying the arc length formula (NCM3.F-TF.1 and NCM3.F-TF.2) In Precalculus, students will extend generating solutions for all trigonometric functions and their reciprocal functions and to all angles. Students will use inverse functions and the following to solve equations: ● the x coordinate of the intersection point of the unit circle and a terminal ray (the cosine of the angle formed by the horizontal axis and the ray). ● The y coordinate of the intersection point of the unit circle and a terminal ray (the sine of the angle formed by the horizontal axis and the ray).	**Indicator:** Solve for x in the interval $[0,2\pi]$: a. For $cos(3x) = \frac{1}{2}$ give exact values. b. For $sin(x) = 0.2$ round to 3 decimal places. *Answers: a.* $x = \left\{\frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9}\right\}$, *b.* $x \approx 0.201$ **Indicator:** Ammunition is fired from a cannon at an initial velocity of 78ft/s and the horizontal distance covered is given by the function $y = 78sin2(\theta) + 15$. What angle should be used to hit a target on the ground that is 89 ft in front of the cannon? (Round to the nearest hundredth) *Answer: 35.79º* **Indicator**: A Ferris wheel is 4 feet off the ground. It has a diameter of 24 feet and rotates once every 60 seconds. Ben sits in a chair that is initially 10 feet above the ground. a. Write a cosine equation to represent the situation. b. When will Ben be 20 feet high during the first minute of his ride? *Answer:* *a.* $y = -12cos(6x + 60) + 16$ *b. at x = 8.2ft, 31.7ft*

Return to: Standards

PC.F.3 Apply properties of trigonometry to solve problems involving all types of triangles. PC.F.3.2 Implement the Law of Sines and the Law of Cosines to solve problems.	
Clarification	**Checking for Understanding**
Students have previously worked with finding the missing sides and angles of *right* triangles using the Pythagorean Theorem and by applying the trigonometric ratios, respectively. In Precalculus, students will use the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles. They should be able to: • distinguish between situations that require the Law of Sines (ASA, AAS, SSA) and situations that require the Law of Cosines (SAS, SSS), • solve for missing side lengths and angles using Law of Sines and Law of Cosines, and • represent real world problems with diagrams of non-right triangles and use them to solve for unknown side lengths and angle measures. ***Note:*** *The ambiguous case for oblique triangles is NOT an expectation in NC Math 4.*	**Indicator:** A plane is flying from city D to city B, which is 115 miles due north. After flying 45 miles, the pilot must change course and fly 15° west of north to avoid a thunderstorm. a. Draw a diagram to represent this situation. b. If the pilot remains on this course for 25 miles, how far will the plane be from city B? c. How many degrees will the pilot need to turn to the right to fly directly to city B?  *Answers:* *a. See diagram to the right.* *b. ≈ 46.3 miles* *c. ≈ 157.02°* **Indicator:** Two fire towers are 21 miles apart. Tower B is due East of Tower A. The rangers at Tower A spot a fire at 43°north of east. The rangers at Tower B spot the same fire 31° north of west. How far is Tower B from the fire? *Answer: ≈ 14.899 miles*

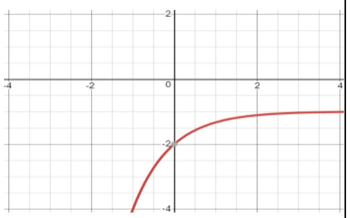
Return to: Standards

PC.F.3 Apply properties of trigonometry to solve problems involving all types of triangles. PC.F.3.3 Implement the Pythagorean identity to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.	
Clarification	**Checking for Understanding**
Grade 8 students study the Pythagorean theorem to find the length of one side of a right triangle given the lengths of two sides. In Math 2 students learn right triangle trigonometry and are familiar with trigonometric ratios. In Precalculus, students are expected to use the Pythagorean identity to find the exact value of $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, $\csc(\theta)$, $\sec(\theta)$, and/or $\cot(\theta)$given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	**Indicator:** Suppose that $cos(\theta) = \frac{1}{3}$and that θ is in the fourth quadrant. Find the exact values for each of the following: a. $sin(\theta)$ b. $tan(\theta)$ c. $csc(\theta)$ d. $sec(\theta)$ e. $cot(\theta)$ *Answers: a.* $-\frac{2\sqrt{2}}{3}$, *b.* $-2\sqrt{2}$, *c.* $-\frac{3\sqrt{2}}{4}$, *d. 3, e.* $-\frac{\sqrt{2}}{4}$

Return to: Standards

PC.F.4 Understand the relationship of algebraic and graphical representations of exponential, logarithmic, rational, power functions, and conic sections to their key features.

PC.F.4.1 Interpret algebraic and graphical representations to determine key features of exponential functions. *Key features include: domain, range, intercepts, intervals where the function is increasing, decreasing, positive or negative, concavity, end behavior, limits, and asymptotes.*

Clarification	Checking for Understanding
In Math 1, students identify and interpret parts of an exponential equation as well as evaluate exponential growth and decay. They also create and graph exponentials. In Math 3, students identify key features including asymptotes and end behavior. In Precalculus, students extend their knowledge of key features to include concavity based on the graph of a function. Students should use limits to describe asymptotes and end behavior. They should also describe limits from the left, right and overall for rational functions.	**Indicator:** Given the function $f(x) = 4^x + 1$, describe the key features. Include domain, intercepts, range, intervals of increasing, decreasing, positive or negative, concavity, end behavior, and asymptotes. *Answer:* *domain: all real numbers* — *range:* $[1, \infty)$ *x-intercept:* $\varnothing$ — *y-intercepts: 2* *increasing: all real numbers* — *decreasing:* $\varnothing$ *positive: all real numbers* — *negative:* $\varnothing$ *concavity up: all real numbers* — *concave down:* $\varnothing$ *end behavior:* $\lim_{x\to\infty} f(x) = \infty$ — *end behavior:* $\lim_{x\to-\infty} f(x) = 1$ *vertical asymptotes: none* — *horizontal asymptotes:* $y = 1$ **Indicator:** Use the given graph to label and describe the key features. Include domain and range, intercepts, intervals of increasing or decreasing, concavity, end behavior, and asymptotes.  *Answer:* *domain: all real numbers* — *range:* $(-\infty, \infty)$ *x-intercept:* $\varnothing$ — *y-intercepts:* -2 *increasing: all real numbers* — *decreasing:* $\varnothing$ *concavity up:* $\varnothing$ — *concave down:* $all\ real\ numbers$ *end behavior:* $\lim_{x\to\infty} f(x) = -1$ — *end behavior:* $\lim_{x\to-\infty} f(x) = -\infty$ *vertical asymptotes: none* — *horizontal asymptotes:* $y = -1$

Return to: Standards

PC.F.4 Understand the relationship of algebraic and graphical representations of exponential, logarithmic, rational, power functions, and conic sections to their key features. PC.F.4.2 Integrate information to build exponential functions to model phenomena involving growth or decay.	
Clarification	**Checking for Understanding**
In Math 1 students build exponential functions from geometric sequences to model growth and decay. They know that the base, $1 + r$, is the growth/decay factor, where r is the growth/decay rate. Students have **NOT** been exposed to continual growth and decay with a base of *e*. Precalculus students should be familiar with exponential functions and recognize situations for which an exponential function is an appropriate model. They are expected to use logarithms to solve exponential equations.	**Indicator:** A new computer cost $1500 but it depreciates in value by about 18% each year. Write a model that indicates the value of the computer at *t* years. *Answer:* $y = 1500(1 - 0.18)^t$ **Indicator**: A biologist is researching a newly discovered species of bacteria. At time $t = 0$ hours, he puts 150 bacteria into what he has determined to be a favorable growth medium. Six hours later, he measures 600 bacteria. Use the formula $N = N_0e^{kt}$,where N_0 is the original population, k is the growth constant and t is time passed. a. Assuming exponential growth, what is the growth constant "k" for the bacteria? (Round k to two decimal places.) b. When will the biologist have a population of 5000 bacterium? (Round to two decimal places) *Answer*: *a.* $k = 0.23$*, b. 15.25 hours*

Return to: Standards

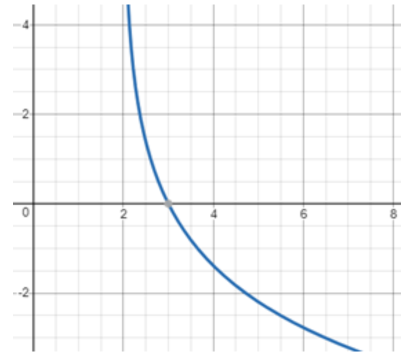
PC.F.4 Understand the relationship of algebraic and graphical representations of exponential, logarithmic, rational, power functions, and conic sections to their key features. PC.F.4.3 Interpret algebraic and graphical representations to determine key features of logarithmic functions. *Key features include: domain, range, intercepts, intervals where the function is increasing, decreasing, positive or negative, concavity, end behavior, continuity, limits, and asymptotes.*	
Clarification	**Checking for Understanding**
Students will extend the key features learned in Math 1-3 to include concavity, horizontal, vertical ***and*** slant asymptotes and limits to describe end behavior and asymptotes for logarithmic functions.	**Indicator:** Given the function $f(x) = log_3 x - 1$, describe the key features. Include domain and range, intercepts, intervals of increasing or decreasing, intervals where the function is positive or negative, concavity, end behavior, and asymptotes. *Answer:* *domain:* $(0, \infty)$ *range:* $(-\infty, \infty)$ *x-intercept: 3* *y-intercept:* $\varnothing$ *increasing:* $(0, \infty)$ *decreasing:* $\varnothing$ *positive:* $(3, \infty)$ *negative:* $(-\infty, 3)$ *concave up:* $\varnothing$ *concave down:* $(0, \infty)$

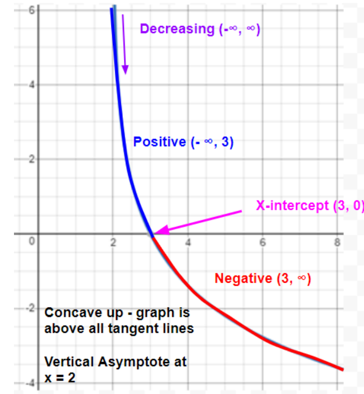
end behavior: $\lim_{x\to\infty} f(x) = \infty$ *end behavior:* $\lim_{x\to 0^+} f(x) = -\infty$

vertical asymptote at $x = 0$ *horizontal asymptote: none*

Indicator: Use the given graph to label and describe the key features. Include intercepts, intervals of increasing or decreasing, Intervals of where the function is positive or negative, concavity and asymptotes.

Answer:





Return to: Standards

PC.F.4 Understand the relationship of algebraic and graphical representations of exponential, logarithmic, rational, power functions, and conic sections to their key features.

PC.F.4.4 Implement graphical and algebraic methods to solve exponential and logarithmic equations in context with support from technology.

Clarification	Checking for Understanding
In Math 3 (NC.M3.F-LE.4) students solve exponential and logarithmic equations algebraically using their understanding of the inverse relationship between the two functions. They are also able to solve graphically with the use of technology or solve using tables. In Precalculus students use properties of logarithms to solve real-world problems with and without technology.	**Indicator:** The function $T(t) = 115e^{-0.043t} + 70$ represents the temperature of a cup of hot chocolate on a cold winter's day, where t is the time after the liquid is served (measured in minutes), and T is temperature measured in degrees Fahrenheit. a. What is the initial temperature of the liquid? b. When will the temperature of the hot chocolate be 85 degrees Fahrenheit? *Answers: a.185ºF, b. approximately 47 min* **Indicator:** The number, n, of college graduates in thousands after t years can be modeled by $n = 48\log_5(t+5)$. Let $t = 0$ represent 1985. a. How many college graduates were there in 2011? b. What year will there be 125,000 college graduates? *Answers: a. 102,000 graduates, b. 2046*

Return to: Standards

PC.F.4 Understand the relationship of algebraic and graphical representations of exponential, logarithmic, rational, power functions, and conic sections to their key features.

PC.F.4.5 Interpret algebraic and graphical representations to determine key features of rational functions. *Key features include: domain, range, intercepts, intervals where the function is increasing, decreasing, positive or negative, concavity, end behavior, continuity, limits, and asymptotes.*

Clarification

Students will extend the key features learned in Math 1-3 to include concavity, horizontal, vertical ***and*** slant asymptotes and limits to describe end behavior and asymptotes.

In Precalculus students are asked to interpret the features of the graph in context. For example: The horizontal asymptote of a drug concentration function indicates the concentration of the drug in the blood over the long-term.

Checking for Understanding

Indicator: Use the properties of logarithms to rewrite the expressions as single logarithms:

a. $log_6 5 + 2log_6 2 + 4log_6 x$

b. $4ln(x) + \frac{1}{2}ln(x) - 2ln(y)$

Answers:

a. $log_6(20x^4)$, b. $ln\frac{x^4\sqrt{x}}{y^2}$

Indicator: Sound is measured in a logarithmic scale using a unit called a *decibel (dB).* The formula for decibel measure is: $D = 10(log\ P - log\ P_0)$, where P is the power or intensity of the sound and P_0 is the weakest sound that the human ear can hear.

If the sound is measured to be 5.2 times the intensity of the weakest sound heard by the human ear, what is the decibel level of the sound?

Answer: 7.16 dB

Indicator: Given the function $f(x) = \frac{2x^2-5x+2}{x^2-4}$ describe the key features. Include domain and range, intercepts, intervals of increasing or decreasing, intervals where the function is positive or negative, concavity, end behavior, continuity, and asymptotes.

Answer:

domain: $(-\infty,-2) \cup (-2,2) \cup (2,\infty)$ — *range:* $(-\infty,2) \cup (2,\infty)$

x-intercept: 3 — *y-intercept:* $\emptyset$

increasing: $(-\infty,-2) \cup (-2,\infty)$ — *decreasing:* $\emptyset$

positive: $(-\infty,-2) \cup (\frac{1}{2},\infty)$ — *negative:* $(-2,\frac{1}{2})$

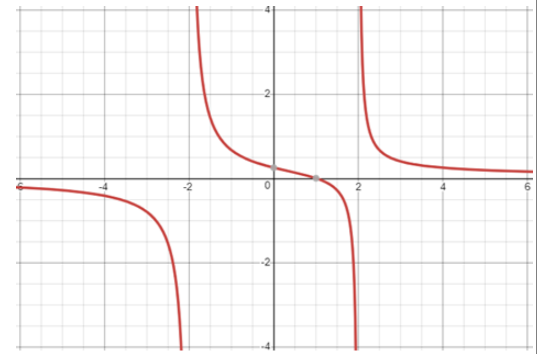
concave up: $(-\infty,-2)$ — *concave down:* $(-2,\infty)$

end behavior: $\lim_{x\to\infty} f(x) = 2$ — *end behavior:* $\lim_{x\to-\infty} f(x) = 2$

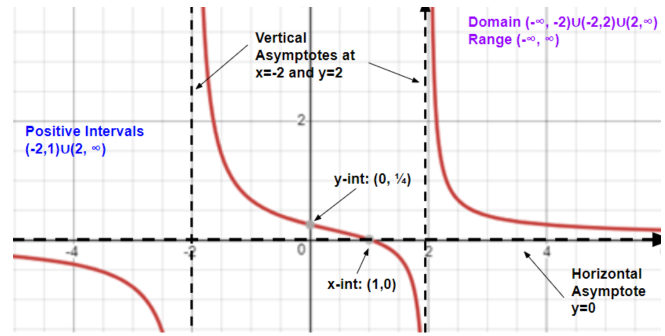
continuity: hole at $(2,\frac{3}{4})$

vertical asymptote: $y = 2$ — *horizontal asymptote:* $x = -2$

Indicator: Use the given graph to label and describe the key features. Include domain and range, intercepts, intervals where the function is positive or negative, and asymptotes.



Answer:



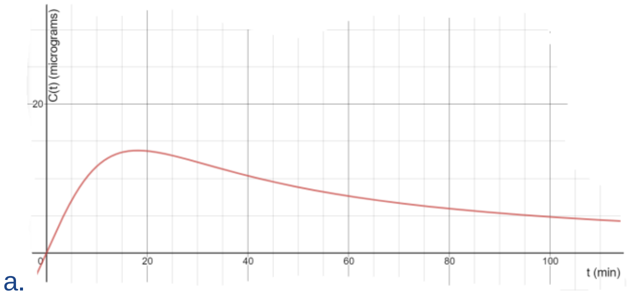
Indicator: Given the function $f(x) = \frac{x^2-4}{x+1}$, describe the key features. Include domain, range, intercepts, and asymptotes.

Answer:

domain: $(-\infty,-1)\cup(-1,\infty)$	*range:* $(-\infty,\infty)$
x-intercept: -2 and 2	*y-intercept: -4*
vertical asymptote at $x=-1$	*horizontal asymptote: none*
	slant asymptote: $y = x - 1$

Indicator: The function: $C(t) = \frac{5t}{0.01t^2+3.3}$ describes the concentration of a drug in the bloodstream over time. In this case, the medication was taken orally. C is measured in micrograms per milliliter and t is measured in minutes.

a. Sketch a graph of the function over the first two hours after the dose is given. Label the axes.

	b. Determine when the maximum amount of the drug is in the body and the amount of that time. c. Explain within the context of the problem the shape of the graph between taking the medication orally ($t = 0$) and the maximum point. What does the shape of the graph communicate between the maximum point and two hours after taking the drug? d. What are the asymptotes of the rational function $C(t) = \frac{5t}{0.01t^2+3.3}$? What is the meaning of the asymptotes within the context of the problem? e. Expand the window of the graph to include negative values for t. Discuss asymptotes. *Answers:*  a. *b. Maximum occurs at 18.17 minutes and that maximum is 13.76 micrograms.* *c. At $t = 0$, the concentration of the drug increases rapidly, and as it approaches 18.17 minutes, the concentration is still increasing but it increases at a slower pace. After $t = 18.17\ min$, the concentration of the drug decreases slowly. At $t = 120\ min$, the concentration of the drug is 4 micrograms.* *d. The horizontal asymptote is $C = 0$. This indicates that the concentration of the drug in the blood system will eventually be close to 0 micrograms.* *e. The graph is asymptotic to the t-axis for negative values of t. $\lim_{t \to -\infty} C(t) = 0$*

Return to: Standards

PC.F.4 Understand the relationship of algebraic and graphical representations of exponential, logarithmic, rational, power functions, and conic sections to their key features.

PC.F.4.6 Implement graphical and algebraic methods to solve optimization problems given rational and polynomial functions in context with support from technology.

Clarification

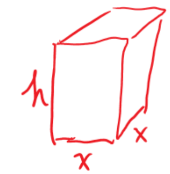
In Math 1 students solve optimization problems involving areas using quadratic functions. In Math 3 students use polynomial functions to express and solve problems involving volume and surface area

In Precalculus, students will create models from descriptions of scenarios in real-world contexts. These models can include rational functions (which goes beyond the models studied in Math 3). The creation of these models can also include the algebraic skills of solving for one variable in terms of another in order to create a function of one variable when the context involves more than one variable as well as utilizing available technology.

Checking for Understanding

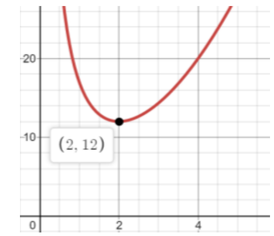
Indicator: A container with a square base, rectangular sides, and no top is to have a volume of $4m^3$.

a. Write a function of one variable for the surface area of the container.
b. Find the dimensions of the box with the minimum surface area.



Answers:

a. Let x be the length of the side of the base of the box and h the height.
   - $4 = x^2h$(volume formula)
   - $\frac{4}{x^2} = h$(solved for h)
   - $S = x^2 + 4xh$(surface area formula)
   - $S = x^2 + 4x(\frac{4}{x^2})$(substitute for h)
   - $S = x^2 + \frac{16}{x}$

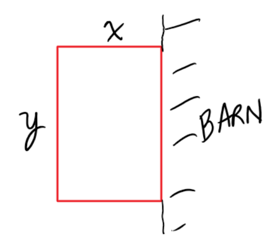


b. Minimum surface area occurs when x = 2 meters, the height of the box is 1 meter.

Indicator: A farmer wants to build a rectangular pen for her goats by enclosing a rectangle of land bordered with a side of the barn as one side of the region and with a fence on the other 3 sides. The farmer has a total of 125 feet of fencing.

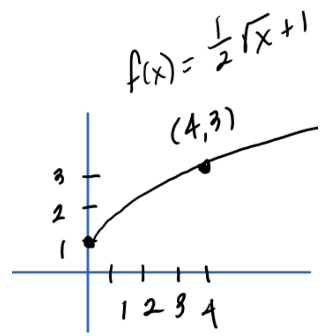
a. Write a function of one variable for the area of the pen.
b. Find the dimensions of the rectangular region of maximum area.

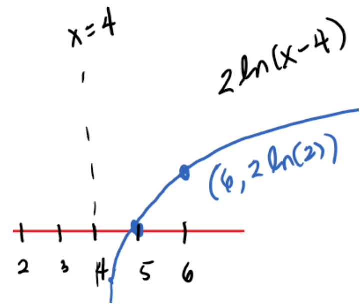
Answers:



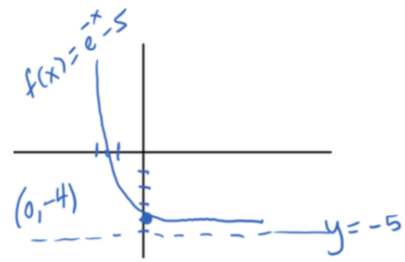
a. Create a model/function for the area of the pen in terms of one variable:
   $2x + y = 125$ and $A = xy$
   Using substitution, $A = x(125 - 2x)$
b. The dimensions of the pen of maximum of area: $x = 31.25ft$ and $y = 62.5ft$

Return to: Standards

<table>
<tr><td colspan="2">PC.F.4 Understand the relationship of algebraic and graphical representations of exponential, logarithmic, rational, power functions, and conic sections to their key features.
PC.F.4.7 Construct graphs of transformations of power, exponential, and logarithmic functions showing key features.</td></tr>
<tr><td>Clarification</td><td>Checking for Understanding</td></tr>
<tr><td>Students will extend the key features learned in Math 1-3 to include concavity, horizontal, vertical and slant asymptotes as well as limits to describe end behavior and asymptotes. Students can solve exponential and log equations to give the exact values of x- and y-intercepts.</td><td>Indicator: Construct a graph for each of the functions without the use of technology. Label and describe the key features of each function. Include: domain, range, intercepts, intervals where the function is increasing or decreasing, intervals where the function is positive or negative, concavity, end behaviors, and asymptotes.
a. $f(x) = \frac{1}{2}\sqrt{x} + 1$
b. $f(x) = 2ln(x - 4)$
c. $f(x) = e^{-x} - 5$
Answers:


a.
• domain: $[0, \infty)$
• range: $[1, \infty)$
• x-intercept: none
• y-intercept: (0,1)
• The function is always increasing (-∞, ∞).
• The function is positive for all x-values in the domain (-∞,∞).
• The function is always concave down.
• $\lim_{x\to\infty} f(x) = \infty$
• $\lim_{x\to-\infty} f(x) = 1$
• No asymptotes


b.
• domain $(4, \infty)$
• range $(-\infty, \infty)$
• x-intercept: (5,0)
• y-intercept: none
• The function always increasing (-∞,∞).
• The function is positive when x is $(5, \infty)$
• The function is negative when x is $(4,5)$
• The function is always concave down
• $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = -\infty$
• vertical asymptote: $x = 4$
• horizontal asymptote: none</td></tr>
</table>

	 c. ● Domain: all real numbers ● Range: $(-5, \infty)$ ● x-intercept: (-ln(5), 0) ● y-intercept: $(0, -4)$ ● The function is always decreasing (-∞, ∞). ● The function is positive when x is $(-\infty, -ln(5))$ ● The function is negative when x is $(-ln(5), \infty)$ ● Function is always concave up ● $\lim_{x\to\infty} f(x) = -5$ ● $\lim_{x\to-\infty} f(x) = \infty$ ● vertical asymptote: none ● horizontal asymptote: $y = -5$

Return to: Standards

PC.F.4 Understand the relationship of algebraic and graphical representations of exponential, logarithmic, rational, power functions, and conic sections to their key features. PC.F.4.8 Identify the conic section (ellipse, hyperbola, parabola) from its algebraic representation in standard form.	
Clarification	**Checking for Understanding**
In Math 1 students are familiar with the relationship between a quadratic equation and a parabola. In Math 3 students demonstrate an understanding of the equation of a circle by writing the equation using the center and radius or by completing the square. In Precalculus, they build on this knowledge to include other conic sections (ellipses and hyperbolas) and describe conic sections in terms of a locus of points.	**Indicator:** Identify the following conic section equations as an ellipse, hyperbola, or parabola. a. $\frac{(x-3)^2}{16} + \frac{y^2}{9} = 1$ b. $\frac{x^2}{16} - y^2 = 1$ c. $x - y^2 = 1$ *Answers: a. ellipse, b. hyperbola, c. parabola*

Return to: Standards

PC.F.4 Understand the relationship of algebraic and graphical representations of exponential, logarithmic, rational, power functions, and conic sections to their key features. PC.F.4.9 Interpret algebraic and graphical representations to determine key features of conic sections (ellipse: center, length of the major and minor axes; hyperbola: vertices, transverse axis; parabola: vertex, axis of symmetry).	
Clarification	**Checking for Understanding**
In Math 1 and 2 students focus on vertical quadratics. In Math 3, students utilize completing the square to write the equation of a circle.	**Formative check:** Given the conic section $4x^2 - 24x - 25y^2 + 250y - 489 = 0$, name the conic and then use completing the square to write in standard form. *Answer: Hyperbola,* $\frac{(y-5)^2}{4} - \frac{(x-3)^2}{25} = 1$

In Precalculus, students extend their knowledge of completing the square to include horizontal parabolas, horizontal and vertical hyperbolas and horizontal and vertical ellipses as well as describe the key features of each (e.g. ellipse: foci, center, length of the major and minor axes; hyperbola: foci, vertices, transverse axis; parabola: focus, vertex, axis of symmetry). Students should be able to graph conic sections and identify key features.	**Indicator:** Based on your identification of conic section equations below, provide the appropriate key features for that conic section. a. $\frac{(x-3)^2}{16}+\frac{y^2}{9}=1$ b. $\frac{x^2}{16}-y^2=1$ c. $x-y^2=1$ *Answers:* *a.* Horizontal Ellipse with center at (3, 0); Major Axis length 8; Minor axis length 6 *b.* Vertical Hyperbola with center at (0,0); Vertices at (0,4) and (0, -4); Transverse Axis length 2 *c.* Horizontal Parabola with vertex at (1,0); Axis of Symmetry: $y=0$

Return to: Standards

PC.F.5 Apply properties of function composition to build new functions from existing functions. PC.F.5.1 Implement algebraic procedures to compose functions.	
Clarification	**Checking for Understanding**
In Math 3, students build new functions using arithmetic operations but do not build new functions using composition. In Precalculus students will recognize composition and be able to apply it. Given two functions, students will be able to create a new function using the operation of composition. They need to reason about domains when two functions are composed. They also need to be able to recognize functions within functions to describe them using composition. In problems involving roots in the denominator, it is not necessary to rationalize the denominator.	**Formative check:** Given $f(x)=\frac{1}{x^2+4}$ and $g(x)=\sqrt{x+1}$. Find $f(g(x))$ and $g(f(x))$. *Answers:* $f(g(x))=\frac{1}{(\sqrt{x+1})^2+4}$ *and* $g(f(x))=\sqrt{\frac{x+5}{x+4}}$ **Indicator:** The price, p, and the quantity sold, x, of a certain product follow the demand equation $x=-5p+100$, where $0\le x\le 20$. The revenue for the product is $R=px$. a. Express revenue, R, as a function of x. b. What price should the company charge to maximize revenue? *Answers:* a. $R(x)=\left(-\frac{x}{5}+20\right)x$*, b. Find the vertex, (50, 500). The quantity sold is 50 units. The maximum revenue is $500. d.*$-\frac{50}{5}+20=p$*, The company should charge $10 per unit.*

Return to: Standards

PC.F.5 Apply properties of function composition to build new functions from existing functions.

PC.F.5.2 Execute a procedure to determine the value of a composite function at a given value using algebraic, graphical, and tabular representations.

Clarification

Students compare key features of two functions using different representations by comparing the properties of the two functions, each with a different representation. However, they do not do anything with composition of functions in Math 3.

In Precalculus students will build an understanding of the fact that the input of the first function produces an output, which then becomes the input for the second function. The output that is produced by this input is the output of the composition. Students will be able to use multiple representations (algebraic, graphical, and tabular) to evaluate the composition of two functions.

Checking for Understanding

Indicator:

x	-2	-1	0	1	2	3	4	5
$f(x)$	5	2	1	2	5	10	17	26

x	-2	-1	0	1	2	3	4	5
$g(x)$	-1	1	3	5	7	9	11	13

Given the two functions above, find the following:

a. $f(g(-2))$
b. $g(f(2))$
c. $(f \circ g)(1)$
d. $(g \circ f)(-1)$

Answers: a. 2, b. 13, c. 26, d. 7

Indicator: Given the two functions, $f(x) = 2x - 3$ and $g(x) = x^2 + 1$, find the following with the given values:

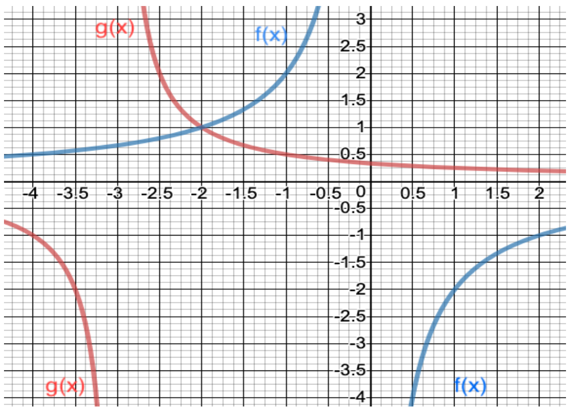
a. $f(g(1))$
b. $g(f(-1))$
c. $(f \circ f)(-3)$
d. $(g \circ g)(0)$

Answers: a. 1, b. 26, c.-21, d. 2

Indicator: Given the two functions in the graph, find the following:

a. $f(g(-2))$
b. $g(f(2))$
c. $(f \circ g)(-4)$
d. $(g \circ f)(1)$

Answers: a. -2, b. 0.5, c. 2, d.1



Return to: Standards

<table>
<tr><th colspan="2">PC.F.5 Apply properties of function composition to build new functions from existing functions.
PC.F.5.3 Implement algebraic methods to find the domain of a composite function.</th></tr>
<tr><th>Clarification</th><th>Checking for Understanding</th></tr>
<tr><td>Students are introduced to the concept of domain (input) and range (output) of linear, exponential, and quadratic functions in Math 1. In Math 2, the concepts of domain and range are extended to square root, inverse variation, and transformed functions. In Math 3, the concept of domain and range is again extended to piecewise, absolute value, polynomials, exponential, rational, and trigonometric functions (sine and cosine).
In Precalculus students will build an understanding of the fact that the input of the first function produces an output, which then becomes the input for the second function. The output that is produced by this input is the output of the composition. Since this is the case, students will be able to find the domain of the composition of two functions by first eliminating any value that cannot be utilized in the domain of the inside function. Secondly, they will set the inside function equal to value(s) that cannot be utilized in the outside function and solve this equation. They must also eliminate this solution from the domain of the composition.</td><td>Formative check: If $f(x) = \frac{1}{x-3}$ and $g(x) = \frac{x+1}{2x}$, find $f(g(x))$ and identify its domain.
Answers: $\frac{2x}{1-5x}$
In the function $g(x)$, $x \neq 0$ and in the function $f(x)$, $x \neq 3$, therefore $g(x) \neq 3$. This means that $\frac{x+1}{2x} \neq 3$
The domain is $(-\infty, 0) \cup \left(0, \frac{1}{5}\right) \cup \left(\frac{1}{5}, \infty\right)$

Indicator: Jim works full time at an electronics store. His weekly salary is $500 per week, plus a 4% commission for any sales over $6000. Jim sells enough this week to earn his commission. If $f(x) = x - 6000$and $g(x) = 0.04x$, find $g(f(x))$, and explain what the composition means. Identify its domain.
Answers: $g(f(x)) = 0.04(x - 6000)$The composition represents the commission Jim earns for one week. The domain of this function is $x \geq 6000$.</td></tr>
</table>

Return to: Standards

<table>
<tr><th colspan="2">PC.F.5 Apply properties of function composition to build new functions from existing functions.
PC.F.5.4 Organize information to build models involving function composition.</th></tr>
<tr><th>Clarification</th><th>Checking for Understanding</th></tr>
<tr><td>Precalculus students should be able to build algebraic models involving function composition of existing models and interpret the meaning in terms of a context. Students are not expected to use tables or graphs to build models.</td><td>Indicator: Suppose a group of students collected and analyzed data and found that the number of hours per week that a college student studied and the student's final grade point average (GPA) could be modeled by$gpa(h) = \sqrt{0.5h + 0.9}$.
In this expression h represents the number of hours studied per week, where h can take on any value from 0 hours to 30 hours. Another group found that the relationship between students' final GPAs and their first salaries after college could be modeled by $sal(g) = 6(2.7g + 3.8)^3 + 10{,}500$, where g represents the GPA and can take on any value from 0 to 4.
a. In the context of the problem, consider the meaning of the composition of these two functions.
b. Write an expression for the composition of the functions $gpa(h)$ and $sal(g)$.
c. Find the domain of the composition.
d. Find the value of $sal(g(25))$and explain its meaning in context.</td></tr>
</table>

	Answers: a. *The composition represents the relationship between the number of hours spent studying and their starting salary.* b. $sal(g(h)) = 6(2.7\sqrt{0.5h+0.9}+3.8)^3 + 10{,}500$ c. *Domain of composition [0,30] given in context* d. $sal(g(h)) = \$25872.86$ *is the starting salary for a student who studies 25 hours per week.*

Return to: Standards

PC.F.5 Apply properties of function composition to build new functions from existing functions. PC.F.5.5 Deconstruct a composite function into two functions.	
Clarification	**Checking for Understanding**
In Math 1 students are taught about different structures of functions when they solve linear equations for y and when they factor polynomials. In Math 2 students expand on this concept when they complete the square and when they transform functions. Math 3 students rewrite exponential functions, polynomial functions, and rational functions using algebraic properties. In Precalculus students will decompose the composition of functions into two functions such that $\left(f \circ g(x)\right) = H$.	**Indicator:** Find functions f and g so that $(f \circ g) = H$. a. $H(x) = (3x - 2)^3$ b. $H(x) = \sqrt[3]{4 + x^2}$ c. $H(x) = \lvert 2x - 5\rvert$ *Answers: a.* $f(x) = x^3$ *and* $g(x) = 3x - 2$, *b.* $f(x) = \sqrt[3]{x}$ *and* $g(x) = 4 + x^2$, *c.* $f(x) = \lvert x\rvert$ *and* $g(x) = 2x - 5$

Return to: Standards

PC.F.5 Apply properties of function composition to build new functions from existing functions. PC.F.5.6 Implement algebraic and graphical methods to find an inverse function of an existing function, restricting domains if necessary.	
Clarification	**Checking for Understanding**
In Math 3 students build an understanding of the inverse relationship between exponential and logarithmic functions, quadratic and square root functions, and linear to linear functions. They determine if an inverse exists by examining tables, graphs, and equations. Students should be able to tell if a function or a portion of the function has an inverse from the above-mentioned representations. If an inverse exists for a linear, quadratic, or exponential function, students find this inverse and represent it using inverse function notation, $f^{-1}(x)$. For quadratic functions students must be able to determine the appropriate domain for the function to have an inverse. In Precalculus students will extend their understanding of the inverse relationship to polynomial functions, as well as the six trigonometric functions. They will build an understanding of a function being one-to-one using the horizontal line test. Students will also build on their ability to restrict the domain of the original function in order for this function to	**Formative check:** a. Find the inverse of the function, $f(x) = \frac{2x+1}{x+3}$. b. State the domain of f. c. State the domain and range of $f^{-1}(x)$. *Answers:* *a.* $f^{-1}(x) = \frac{3x-1}{2-x}$ *b. Domain of f:* $(-\infty,-3) \cup (-3,\infty)$ *c. Domain of* f^{-1}: $(-\infty,2) \cup (2,\infty)$ *and range of* f^{-1}: $(-\infty,-3) \cup (-3,\infty)$ **Formative check:** The function $C(x) = \frac{5}{9}(x - 32)$ converts Farenheit temperatures to Celsius temperatures. a. Find the inverse. b. Interpret its meaning. Answers: *a.* $C^{-1}(x) = \frac{9}{5}x + 32$,

be one-to-one. Students will find an inverse graphically and algebraically.

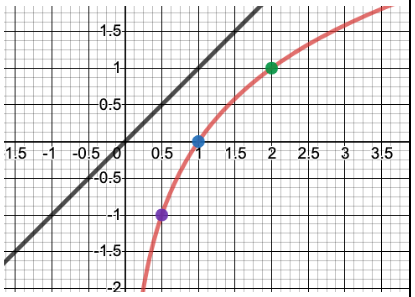
b. *The function converts temperature measurements from Celsius to Fahrenheit.*

Formative check: A rock is thrown into a puddle creating a ripple in the water that travels outward at 30cm/s.

a. Write a rule that models the radius as a function of time.
b. Find the inverse of the function.
c. Describe what the inverse represents.

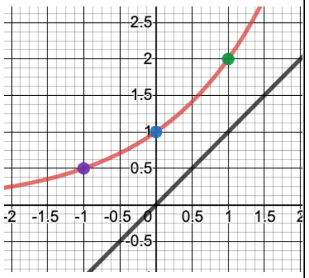
Answers:
a. $r(t) = 30t$, b. $T(r) = \frac{r}{30}$, *c. The inverse tells us how long ago the rock was thrown based on the largest radius.*

Indicator: For the function graphed below: (Note: this is the same example as the first Formative check with the Precalculus extension added.)

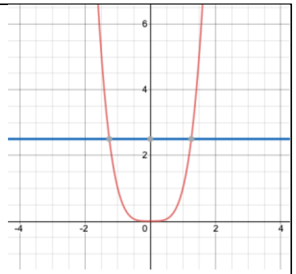
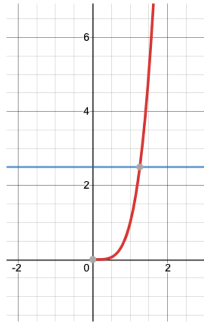


a. Graph the inverse.
b. Is the inverse a function?
c. Explain how you know the new graph is the inverse of the original graph?
d. Explain the relationship between the two graphs

Answers:



a. *Graph*
b. *The inverse is a function because the original graph passes the horizontal line test. Also, the inverse passes the vertical line test.*
c. *The new graph is the inverse of the original graph because the x and y-values for each ordered pair of the original function have been interchanged. This fact makes the new function a reflection of the original function over the line y = x.*
d. *This tells us that the original function is one-to-one.*

	Indicator: The function $f(x) = x^4$is not one-to-one. a. Find a suitable domain restriction for f that will result in a one-to-one function. b. Find the inverse of f.  *Answers:*  *a. One possible restriction is $f(x) = x^4$, when $\boldsymbol{x} \geq 0$. This will make f(x) one-to-one.* *b. $f^{-1}(x) = x^{1/4}$, when $\boldsymbol{x} \geq 0$.* **Indicator:** A large set of dice hanging from the rear-view mirror of a car has a side that measures 3.5 cm. a. Write a rule that models the Volume of the die as a function of its side length. b. Find the inverse of the function. c. Describe what the inverse represents. *Answers:* *a. $V(S) = S^3$* *b. $S(V) = \sqrt[3]{V}$* *c. The inverse tells us the length of the side of a cube given the volume.*

Return to: Standards

PC.F.5 Apply properties of function composition to build new functions from existing functions. PC.F.5.7 Use composition to determine if one function is the inverse of another function.	
Clarification	**Checking for Understanding**
In Math 3 students build an understanding of the inverse relationship between exponential and logarithmic functions, quadratic and square root functions, and linear to linear functions. They determine if an inverse exists by examining tables, graphs, and equations. If an inverse exists for a linear, quadratic, or exponential	**Formative check:** Find the inverse of the one-to-one function, $f(x) = 2\sqrt[3]{x}$, verify your answer using composition. *Answer:* $f^{-1}(x) = \frac{x^3}{8}$ *Verification:* $f^{-1}\big(f(x)\big) = \frac{\left(2\sqrt[3]{x}\right)^3}{8} = \frac{8x}{8} = x$

function, students find this inverse and represent it using inverse function notation, $f^{-1}(x)$. In Precalculus students will extend their understanding of the inverse relationship to polynomial and rational functions. Students will understand that a function whose inverse is also a function is called a one-to-one function. Students will also use composition to verify that two functions are inverses of each other.	$f\left(f^{-1}(x)\right) = 2\sqrt[3]{\frac{x^3}{8}} = 2\left(\frac{x}{2}\right) = x$ **Indicator:** A student was asked to find the inverse of $g(x) = 4x - 5$. The student's work is shown below. Use composition to show that the student is incorrect. Explain why the student is incorrect and what should have been done to arrive at the correct answer. Student's work: The function $g(x) = 4x - 5$involves two operations, multiplying by 4 and subtracting 5. So, the inverse would include dividing by 4 and adding 5. The inverse function is $g^{-1}(x) = \frac{x}{4} + 5$. *Answer: Composing the function with what the student believes is the inverse yields:* $g(g^{-1}(x)) = 4(\frac{x}{4} + 5) = x + 20$ $g^{-1}(g(x)) = \frac{4x-5}{4} + 5 = x - \frac{5}{4} + 5 = x + 3\frac{3}{4}$ *Since* $g(g^{-1}(x)) \neq x$ *and* $g^{-1}(g(x)) \neq x$ *then the student's work is incorrect. The student recognized that the inverse function uses inverse operations but failed to apply these operations in reverse order. In other words, switch the* x *and* y *and solve for* y. $x = 4y - 5$ $g^{-1}(x) = \frac{x+5}{4}$

Return to: Standards

PC.F.6 Apply mathematical reasoning to build recursive functions and solve problems. PC.F.6.1 Use algebraic representations to build recursive functions.	
Clarification	**Checking for Understanding**
In Math 1 students translated between recursive and explicit arithmetic and geometric sequences. In Precalculus students analyze contextual situations and write recursive equations that go beyond geometric or arithmetic relationships. Series are not an expectation.	**Indicator:** Shaneka has a headache and decides to take a 200 mg ibuprofen tablet for her pain. The drug is absorbed into her system and stays in her system until the drug is metabolized and filtered out by her liver and kidneys. Every four hours, Shaneka's body removes 67% of the ibuprofen that was in her body at the beginning of that four-hour period. a. Write a set of recursive equations that represent the amount of ibuprofen in Shaneka's body at the end of each four-hour period. b. If Shaneka takes 400 mg every four hours after her initial dose of 200 mg, write a set of recursive equations that represent the amount of ibuprofen in Shaneka's body at the end of each four-hour period.

Answers:

a. $D_0 = 200 \;\; D_n = D_{n-1} - 0.67D_{n-1}$ *or* $D_n = 0.33D_{n-1}$*where* n *is measured in 4-hour periods.*

b. $D_0 = 200 \;\; D_n = D_{n-1} - 0.67D_{n-1} + 400$ *or* $D_n = 0.33D_{n-1} + 400$*where* n *is measured in 4-hour periods.*

Indicator: A population of fish in a pond decreases each month by 20%. The pond is restocked each month by adding 100 fish. If the initial population of fish is 250.

a. Write a set of recursive equations that gives the population of the fish pond over time.
b. What is the population of the fish after 1 year?
c. What is the fish population in the long run?

Answers:

a. $F_0 = 250 \;\; F_n = F_{n-1} - 0.20F_{n-1} + 100$ *or* $F_n = 0.9F_{n-1} + 100$*where* n *is measured in months.*

b. *The fish population is approximately 483 fish after 12 months.*

c. *In the long run the fish population will stabilize to 500 fish.*

Return to: Standards

PC.F.6 Apply mathematical reasoning to build recursive functions and solve problems.

PC.F.6.2 Construct a recursive function for a sequence represented numerically.

Clarification	Checking for Understanding
This extends what students learn in Math 1, recognizing recursively defined relationships from tables. In Precalculus, students create recursive equations that go beyond arithmetic and geometric sequences. Series are not an expectation.	**Indicator:** Given the values shown to the right, write a recursive function that represents the sequence of values in the table. *Answer: Noticing that the new value is double the previous value, we can write:* $x_n = 2x_{n-1}$ **Indicator:** Given the values shown, write a recursive function that represents the sequence of values in the table. *Answer: Consider the differences between consecutive terms, we see that these are 2, 1, 0.5, 0.25. Each difference is ½ that of the previous difference. The sequence "starts" at 4 so we can add ½ of the previous term to 4 which leads us to the recursive equation:* $x_n = \frac{1}{2}x_{n-1} + 4$

1	20
2	40
3	80
4	160
5	320

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PC.F.7 Apply mathematical reasoning to build parametric functions and solve problems.
PC.F.7.1 Implement algebraic methods to write parametric equations in context.

Clarification	Checking for Understanding
In Math 1, students create linear and quadratic equations to model situations in context and use them to solve problems. In Precalculus, students use parametric equations to solve contextual problems given parametric equations and build linear and quadratic models for each of the horizontal and vertical positions of objects in contextual problems and use those equations to solve problems.	**Indicator:** The Sea Queen leaves the port at Nassau at 7:00 PM. She sails due east at 15 mph. The Ocean Princess leaves a small island that is 27 miles north of Nassau at the same time. She sails due south at 22 mph. Assuming the ships continue in the same directions at the same speeds, a. Write parametric equations that model the paths of the two ships as they would appear on a radar screen located at Nassau. b. Let d represent the distance between the two ships. Express d as a function of t, the number of hours elapsed since 7:00 PM. *Answer: Assume the port of Nassau is located at (0,0)* *a. Sea Queen's position Parametric Equations:* $x(t) = 15t \quad y(t) = 0$ *Ocean Princess' Parametric Equations:* $x(t) = 0 \quad y(t) = 27 - 22t$ *b. Distance function:* $d(t) = \sqrt{(15t)^2 + (27 - 22t)^2}$ **Indicator:** The horizontal position of a projectile is given by the equation:$x(t) = 58t$ The vertical position of the projectile is given by the equation: $y(t) = -16t^2 + 6$ where t is measured in seconds and x and y are measured in feet. a. Find the position of the projectile at time t = 0.2 seconds. Round your answers to 2 decimal places. b. A 4 foot fence is located 25 feet horizontally from the projectile's initial position. Will the projectile clear the fence? *Answer:* *a.* $x(0.2) = 11.73$*feet and* $y(0.2) - 5.61$*feet* *b. No, since the projectile's horizontal position is 25 when* $t = 0.43$ *seconds, and* $y(0.43) = 3.04$ *feet which is less than 4 feet.*

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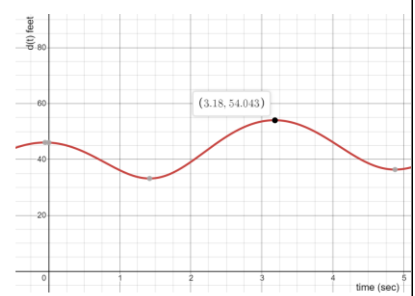
<table>
<tr><td colspan="2">PC.F.7 Apply mathematical reasoning to build parametric functions and solve problems.
PC.F.7.2 Implement technology to solve contextual problems involving parametric equations.</td></tr>
<tr><td>Clarification</td><td>Checking for Understanding</td></tr>
<tr><td>In Math 3, students use technology for more complicated functions to find extreme values and analyze other function behavior.
In Precalculus, students use parametric equations to solve contextual problems given parametric equations and create parametric equations as models for more complicated functions in context and use those models and technology to solve problems.</td><td>Indicator: The Sea Queen leaves the port at Nassau at 7:00 PM. She sails due east at 15 mph. The Ocean Princess leaves a small island \that is 27 miles north of Nassau at the same time. She sails due south at 22 mph. Assuming the ships continue in the same directions at the same speeds, write parametric equations that model the paths of the two ships as they would appear on a radar screen
located at Nassau.
Let d represent the distance between the two ships.
a. Express d as a function of t, the number of hours elapsed since 7:00 PM.
b. Find the distance between the two ships 2 hours after they leave port. Round your answer to 2 decimal places.
c. Find the minimum distance between the two ships. Round your answer to two decimal places.
Answer:
a. $d(t) = \sqrt{(15t)^2 + (27 - 22t)^2}$, b. 34.48 mi, c. 15.21 mi

Indicator: The position of competitive ice skater is given by the set of parametric equations:
$x(t) = 50cos(t)$ and $y(t) = 35sin(t)$ where t is measured in seconds and x and y are measured in feet. A judge is located at the point (4,1.6). Find the maximum distance between the judge and the skater.
Answer:
The distance between the skater and the judge is given by
$d(t) =$
$\sqrt{(50cos(t) - 4)^2 + (35sin(t) - 1.6)^2}$
Maximum value of d is 54.04 feet. See graph.

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